Chapter 6 : Algebraic Expressions and Identities

ANSWER KEYS

EXERCISE 6.1

1. (*i*) $5ab^2c - 4ab + 1$

Term	Coefficient
$5ab^2c$	5
$-4ab$	-4

Constant term = 1

(*ii*) $3 + a^2 - 5b^2$ or $a^2 - 5b^2 + 3$

Term	Coefficient
a^2	1
$-5b^2$	-5

Constant term = 3

(*iii*) $6x^2y^2 - 5x^2z^2 + 7$

Term	Coefficient
$6x^2y^2$	6
$-5x^2z^2$	-5

Constant term = 7

(*iv*) $-ab + 2a^2b^2 + 5$

Term	Coefficient
$-ab$	-1
$2a^2b^2$	2

Constant term = 5

2. **Monomial:** (*iv*) $3a$

Binomial: (*i*) $2x + y$, (*v*) $7ab - 5a$ (*vi*) $x^2 - 5$

Trinomial: (*ii*) $a^2 + a - 5$, (*iii*) $5x - 2y + 3xy$

3. (*i*) $6a$, $13b^2$, $-2a$, $2a^2$, $-b^2$, bc

Like terms: $6a$, $-2a$ and $13b^2$, $-b^2$

(*ii*) $4x^2$, $-5x$, 6, $7x$, $-2x^2$, -3

Like terms: $-5x$, $7x$; $4x^2$, $-2x^2$ and 6, -3

(*iii*) $2a^2b^2$, $6a^2b$, $2ab^2$, $-3a^2b^2$, $7ab^2$, $-9a^2b$

Like terms: $2a^2b^2$, $-3a^2b^2$; $6a^2b$, $-9a^2b$, $2ab^2$, $7ab^2$

4. (*i*) x^2, $2x^2$, $-6x^2$

$$x^2 + 2x^2 + (-6x^2) = x^2 + 2x^2 - 6x^2$$
$$= -3x^2$$

(*ii*) $xy - yz$, $yz + 2xy$

$$(xy - yz) + (yz + 2xy) = xy - yz + yz + 2xy$$
$$= (xy + 2xy) + (-yz + yz)$$
$$= 3xy + 0 = 3xy$$

(*iii*) $3x^2y^2 - 4xy + 5$, $2x^2y^2 + 3xy - 7$

$$(3x^2y^2 - 4xy + 5) + (2x^2y^2 + 3xy - 7)$$
$$= (3x^2y^2 + 2x^2y^2) + (-4xy + 3xy) + (5 - 7)$$

(Collecting like terms together)

$$= 5x^2y^2 - xy - 2$$

(*iv*) $7xy + 4xz - 2yz$, $2xy - 3xz + 6yz$

$$(7xy + 4xz - 2yz) + (2xy - 3xz + 6yz)$$
$$= (7xy + 2xy) + (4xz - 3xz) + (-2yz + 6yz)$$
$$= 9xy + xz + 4yz$$

5. (*i*) $6x^2$ from $-2x^2$

$$-2x^2 - 6x^2 = -8x^2$$

(*ii*) $4a + 5ab - 7$ from $3a - 2ab - 6$

$$3a - 2ab - 6 - (4a + 5ab - 7)$$
$$= (3a - 4a) + (-2ab - 5ab) + (-6 + 7)$$
$$= -a - 7ab + 1$$

(*iii*) $6x^2y^2 - 7x^2y + 4xy^2 + 11$ from $5x^2y^2 - 3x^2y + 5xy^2 + 7$

$$5x^2y^2 - 3x^2y + 5xy^2 + 7 - 6x^2y^2 + 7x^2y - 4xy^2 - 11$$
$$= (5x^2y^2 - 6x^2y^2) + (-3x^2y + 7x^2y) + (5xy^2 - 4xy^2) + 7 - 11$$
$$= -x^2y^2 + 4x^2y + xy^2 - 4$$

(*iv*) $7a^3b - 4a^2b^2 + 9$ from $3a^3b + 2a^2b^2 + 7$

$$3a^3b + 2a^2b^2 + 7 - (7a^3b - 4a^2b^2 + 9)$$
$$= 3a^3b + 2a^2b^2 + 7 - 7a^3b + 4a^2b^2 - 9$$
$$= (3a^3b - 7a^3b) + (2a^2b^2 + 4a^2b^2) + 7 - 9$$
$$= -4a^3b + 6a^2b^2 - 2$$

6. $2xy + 3yz - 6xz$, $4xy - 2yz + 7xz$ and $-xy - 2yz + 4xz$

$$(2xy + 3yz - 6xz) + (4xy - 2yz + 7xz) + (-xy - 2yz + 4xz)$$
$$= (2xy + 4xy - xy) + (3yz - 2yz - 2yz) + (-6xz + 7xz + 4xz)$$
$$= 5xy - yz + 5xz$$

7. $\frac{1}{3}x^2 + \frac{1}{2}x + 2 - \left(x^2 - \frac{5}{2}x + 7\right)$

$$= \frac{1}{3}x^2 + \frac{1}{2}x + 2 - x^2 + \frac{5}{2}x - 7$$
$$= \left(\frac{1}{3} - 1\right)x^2 + \left(\frac{1}{2} + \frac{5}{2}\right)x - 5$$
$$= \frac{-2}{3}x^2 + 3x - 5$$

8. Addition of $2x^2 - 5x^2y^2 + 2xy$ and $6xy + 4x^2 - 7$.

$2x^2 - 5x^2y^2 + 2xy + 6xy + 4x^2 - 7$

$= (2x^2 + 4x^2) - 5x^2y^2 + (2xy + 6xy) - 7$

$= 6x^2 - 5x^2y^2 + 8xy - 7$

Now,

$$\begin{array}{rrrr} x^2 & +7x^2y^2 & -xy & \\ 6x^2 & -5x^2y^2 & +8xy & -7 \\ - & + & - & + \\ \hline -5x^2 & +12x^2y^2 & -9xy & +7 \\ \hline \end{array}$$

Hence, required result $= -5x^2 + 12x^2y^2 - 9xy + 7$

9. Required result $= 2a^2 + 7ab + 2c^2 - (5a^2 - 2ab + c^2)$

$= 2a^2 + 7ab + 2c^2 - 5a^2 + 2ab - c^2$

$= (2a^2 - 5a^2) + (7ab + 2ab) + (2c^2 - c^2)$

$= -3a^2 + 9ab + c^2$

Hence, required result $= -3a^2 + 9ab + c^2$

10. Sum of $2x - y + 7$ and $2y - 10$

$= (2x - y + 7) + (2y - 10)$

$= 2x + y - 3$

Now,

$$\begin{array}{rrr} 2x & +y & -3 \\ 4x & -2y & -11 \\ - & + & + \\ \hline -2x & +3y & +8 \\ \hline \end{array}$$

(On subtracting)

Hence, required result $= -2x + 3y + 8$

11. Sum of $7 + 3x$ and $6 - 2x + x^2$

$= 7 + 3x + 6 - 2x + x^2$

$= x^2 + x + 13$

And sum of $2x^2 - 3x$ and $2x^2 + 3x - 9$

$= 2x^2 - 3x + 2x^2 + 3x - 9$

$= 4x^2 + 0 - 9$

$= 4x^2 - 9$

Now,

$$\begin{array}{rrr} x^2 & +x & +13 \\ +4x^2 & & -9 \\ - & & + \\ \hline -3x^2 & +x & +22 \\ \hline \end{array}$$

(On subtracting)

Hence, required result $= -3x^2 + x + 22$

12. Required expression $= 3a - 7b - 10 - (-2a - 7b + 9)$

$$\begin{array}{rrr} 3a & +7b & -10 \\ -2a & -7b & +9 \\ + & + & - \\ \hline 5a & +14b & -19 \\ \hline \end{array}$$

(On subtracting)

Hence, required expression $= 5a + 14b - 19$

13. When $m = -2$

(*i*) $3m - 7 = 3 \times (-2) - 7$

$= -6 - 7 = -13$

(*ii*) $2m^2 + 5m - 3 = 2 \times (-2)^2 + 5(-2) - 3$

$= 8 - 10 - 3 = -5$

(*iii*) $3m^3 + 2m^2 - 3m + 5 = 3(-2)^3 + 2(-2)^2 - 3(-2) + 5$

$= 3 \times (-8) + 2 \times 4 + 6 + 5$

$= -24 + 8 + 11 = -5$

14. $3x^2 - x - n = 6$

Substituting $x = 1$ in the given expression, we get

$\Rightarrow \quad 3 \times (1)^2 - (1) - n = 6$

$\Rightarrow \quad 3 - 1 - n = 6$

$\Rightarrow \quad 2 - n = 6$

$\Rightarrow \quad n = -6 + 2$

$\boxed{x = -4}$

Hence, $n = -4$

15. The required expression $= 5x^2 + 4y^2 + 10xy - (2x^2 + 6y^2 + 9xy)$

$$\begin{array}{rrr} 5x^2 & +4y^2 & +10xy \\ 2x^2 & +6y^2 & +9xy \\ - & - & - \\ \hline +3x^2 & -2y^2 & +xy \\ \hline \end{array}$$

Hence, the required expression $= 3x^2 - 2y^2 + xy$

EXERCISE 6.2

1. (*i*) $(2, 4x) = 2 \times 4x = 8x$

(*ii*) $(-3x, 2x) = -3x \times 2x = -6x^2$

(*iii*) $(-7p, 3p^2q) = (-7 \times 3)p^3q = -21\, p^3q$

(*iv*) $(3m^2, -2m^3) = [3 \times (-2)]m^5$

$= -6m^5$

(*v*) $(6x^3, 0) = (6 \times 0)x^3 = 0$

(*vi*) $(15xy, 2xy) = (15 \times 2)x^2y^2 = 30x^2y^2$

2. (*i*) Length $= x^2$, Breadth $= x^3$

Area of a rectangle = length × breadth

$= x^2 \times x^3$

$= x^{2+3} = x^5$

(*ii*) Length $= 2a^2b$, Breadth $= 3ab$

Area of a rectangle = length × breadth

$= (2a^2b) \times (3ab)$

$= 6a^3b^2$

(*iii*) Length $= 6m^2n^2p$, Breadth $= 2mnp^2$

Area of a rectangle = length × breadth

$= 6m^2n^2p \times 2mnp^2$

$= 12m^3n^3p^3$

(*iv*) Length $= 3ab$, Breadth $= 5bc$

Area of a rectangle = length × breadth

$= (3ab) \times (5bc)$

$= 15ab^2c$

(*v*) Length $= 3a$, Breadth $= 2ab$

Area of a rectangle = length × breadth

$= 3a \times 2ab$

$= 6a^2b$

(*vi*) Length = $5xy^3$, Breadth = $\frac{2}{15}x^2y$

Area of a rectangle = length × breadth

$$= (5xy^3) \times \left(\frac{2}{15}x^2y\right)$$

$$= \frac{2}{3}x^3y^4$$

3. (*i*) Product of xy, yz and $zx = xy \times yz \times zx$

$$= x^2y^2z^2$$

(*ii*) Product of a^2b, $-2a^3b^2$ and $ab = -2a^{2+3+1} \cdot b^{1+2+1}$

$$= -2a^6b^4$$

(*iii*) $4mn^2, -2m^2n^2, -3mn^3 = [4 \times (-2) \times (-3)]m^4n^7$

$$= 24m^4n^7$$

(*iv*) $2a^3b^3, 3ab^2, -2a^2b = 2a^3b^3 \times 3ab^2 \times (-2a^2b)$

$$= [2 \times 3 \times (-2)]a^6b^6 = -12a^6b^6$$

(*v*) $2xy^2, 3x^2z, xyz$

$(2 \times 3 \times 1)x^4y^3z^2 = 6x^4y^3z^2$

4. $\because$ Volume of box = length × breadth × height

(*i*) Volume of box $= 7x \times 5x \times 2x^2$

$= (7 \times 5 \times 2)x^4 = 70x^4$

(*ii*) Volume of box $= 4x^2 \times 3xy \times 3x$

$= (4 \times 3 \times 3)x^4y = 36x^4y$

(*iii*) Volume of box $= 6p \times 3q \times 2r$

$= 36pqr$

(*iv*) Volume of box $= 4a^2 \times 5b^2 \times 2ab$

$= (4 \times 5 \times 2)a^3b^3 = 40a^3b^3$

5. (*i*) $(-7ab) \times (2a^2bc) \times (4abc^2) = (-7 \times 2 \times 4)a^4b^3c^3$

$$= -56a^4b^3c^3$$

(*ii*) $\left(\frac{1}{2}x^2y\right)\times\left(\frac{2}{3}xy^3\right)\times 3xy = \left(\frac{1}{2}\times\frac{2}{3}\times 3\right)x^4y^5$

$$= 1 \times x^4y^5 = x^4y^5$$

(*iii*) $\left(\frac{3}{2}x^3y^2\right)\times\left(-\frac{2}{3}xy\right)\times(2xy^3)$

$$= \left[\frac{3}{2}\times\left(-\frac{2}{3}\right)\times 2\right]x^5y^6 = -2x^5y^6$$

(*iv*) $(2a^2b^2)\times\left(\frac{-7}{2}ab\right)\times(3b) = \left[2\times\left(\frac{-7}{2}\right)\times 3\right]a^3b^4$

$$= -21a^3b^4$$

6. $(x^2y) \times (-2xyz) \times (xy^3) = -2x^4y^5z$

To verify: if $x = 2$, $y = 1$, $z = 3$

L.H.S. $= (x^2y) \times (-2xyz) \times (xy^3)$

$= [(2^2) \times 1] \times [-2 \times 2 \times 1 \times 3] \times [2 \times 1^3]$

$= 4 \times (-12) \times 2 = -96$

R.H.S. $= -2x^4y^5z$

$= -2 \times (2)^4 \times (1)^5 \times 3$

$= -2 \times 16 \times 3 = -96$

Hence, L.H.S. = R.H.S.

7. $(3x^2y^2) \times (-5xyz) \times (-x^2y)$

$= [3 \times (-5) \times (-1)]x^5y^4z$

$= 15x^5y^4z$

To verify: if $x = 2$, $y = 1$, $z = 1$

L.H.S. $= (3x^2y^2) \times (-5xyz) \times (-x^2y)$

$= (3 \times 2^2 \times 1^2) \times (-5 \times 2 \times 1 \times 1) \times (-2^2 \times 1)$

$= (3 \times 4 \times 1) \times (-5 \times 2) \times (-4 \times 1)$

$= 12 \times (-10) \times (-4)$

$= 480$

R.H.S. $= 15x^5y^4z$

$= 15 \times (2)^5 \times (1)^4 \times 1$

$= 15 \times 32 \times 1$

$= 480$

$\therefore$ L.H.S. = R.H.S.

8. $(pqr) \times (pq^3r) \times (p^2r) = (p \times p \times p^2) \times (q \times q^3) \times (r \times r \times r)$

$= p^4q^4r^3$

To verify: $p = -1$, $q = 2$, $r = -2$

L.H.S. $= (pqr) \times (pq^3r) \times (p^2r)$

$= [(-1) \times 2 \times (-2)] \times [(-1) \times (2)^3 \times (-2)] \times [(-1)^2 \times (-2)]$

$= 4 \times 16 \times (-2) = -128$

R.H.S. $= p^4q^4r^3$

$= (-1)^4 \times (2)^4 \times (-2)^3$

$= -1 \times 16 \times 8 = -128$

$\therefore$ L.H.S. = R.H.S.

9. $\left(\frac{1}{2}p^3q^6\right)\times\left(-\frac{2}{3}p^4q\right)(pq^2)$

$$= \left[\frac{1}{2}\times\left(-\frac{2}{3}\right)\right]p^8q^9$$

$$= -\frac{1}{3}p^8q^9$$

if $p = 2$, $q = -1$. Therefore,

$$= -\frac{1}{3}\times(2)^8\times(-1)^9$$

$$= -\frac{1}{3}\times 256\times(-1)$$

$$= \frac{256}{3}$$

10.

	xy	x^2y	xy^2	x^3y
x	x^2y	$\boldsymbol{x^3y}$	x^2y^2	$\boldsymbol{x^4y}$
x^2	$\boldsymbol{x^3y}$	x^4y	$\boldsymbol{x^3y^2}$	$\boldsymbol{x^5y}$
x^3	$\boldsymbol{x^4y}$	$\boldsymbol{x^5y}$	x^4y^2	x^6y
xy	x^2y^2	$\boldsymbol{x^3y^2}$	x^2y^3	x^4y^2

EXERCISE 6.3

1. (*i*) $3a^2(2a + 3b) = 6a^3 + 9a^2b$

(*ii*) $2a^3(3a - b) = 6a^4 - 2a^3b$

(*iii*) $7a(2a + 5b) = 14a^2 + 35ab$

(*iv*) $-5x(4x - 3b) = -20x^2 + 15bx$

(*v*) $\frac{2}{3}x(x^2y + xy) = \frac{2}{3}x^3y + \frac{2}{3}x^2y$

(*vi*) $ab(a^3 - b^3) = a^4b - ab^4$

(*vii*) $2.5x(10x^2 + 100y) = 25x^3 + 250xy$

(*viii*) $\frac{2}{3}xy(x^2y - xy^2) = \frac{2}{3}x^3y^2 - \frac{2}{3}x^2y^3$

(*ix*) $\frac{2}{5}x(x^3 + y^2 - z^2) = \frac{2}{5}x^4 + \frac{2}{5}xy^2 - \frac{2}{5}xz^2$

(*x*) $2.1x(2.1x - 3y) = 4.41x^2 - 6.3xy$

2. (*i*) $6x(x - 3) + x(2 - 5x) + 3x^2$

$= 6x^2 - 18x + 2x - 5x^2 + 3x^2$

$= (6x^2 - 5x^2 + 3x^2) + (-18x + 2x)$

$= 4x^2 - 6x$

(*ii*) $x(2x - xy) - 2y(3y - 2x^2y) + x^2(1 + y^2)$

$= 2x^2 - x^2y - 6y^2 + 4x^2y^2 + x^2 + x^2y^2$

$= (2x^2 + x^2) + (x^2y^2 + 4x^2y^2)$

$+ (-6y^2 + y^2)$

$= 3x^2 + 5x^2y^2 - 6y^2 - x^2y$

$= 3x^2 - x^2y - 6y^2 + 5x^2y^2$

(*iii*) $a^2(b^2 - 2a) - 3b(b - 2a^2b) + a^2(3 - 2b^2)$

$= a^2b^2 - 2a^3 - 3b^2 + 6a^2b^2 + 3a^2 - 2a^2b^2$

$= (a^2b^2 + 6a^2b^2 - 2a^2b^2) - 2a^3 - 3b^2 + 3a^2$

$= 5a^2b^2 - 2a^3 - 3b^2 + 3a^2$

$= -2a^3 + 3a^2 - 3b^2 + 5a^2b^2$

(*iv*) $4xy(y - x) - 3y^2(x^2 - x) - 5x^2(y - y^2)$

$= 4xy^2 - 4x^2y - 3y^2x^2 + 3xy^2$

$- 5x^2y + 5x^2y^2$

$= (4xy^2 + 3x^2y) + (-4x^2y - 5x^2y)$

$+ (-3y^2x^2 + 5x^2y^2)$

$= 7x^2y^2 - 9x^2y + 2x^2y^2$

$= 7x^2y^2 - 9x^2y + 2x^2y^2$

(*v*) $a(b - c) + c(a - b) + b(c - a)$

$= ab - bc + ca - cb + bc - ab$

$= (ab - ab) + (-ac + ac) + (-bc + bc)$

$= 0 + 0 + 0 = 0$

(*vi*) $a^3(2b - a^2) - 2b(a^3 - a^2) + 3a^2(a^3 - b)$

$= 2a^3b - a^5 - 2a^3b + 2a^2b + 3a^5 - 3a^2b$

$= (2a^3b - 2a^3b) + (-a^5 + 3a^5) + (2a^2b - 3a^2b)$

$= 0 + 2a^5 - a^2b$

$= 2a^5 - a^2b$

3. Product of $3x^3y^2$ and $(2x - 3y)$

$= 3x^3y^2 \times (2x - 3y)$

$= 6x^4y^2 - 9x^3y^3$

To verify: if $x = -1$, $y = 2$

L.H.S. $= 3x^3y^2(2x - 3y)$

$= 3(-1)^3(2)^2[2 \times (-1) - 3 \times 2]$

$= -12 \times [-2 - 6]$

$= -12 \times (-8) = 96$

R.H.S. $= 6x^4y^2 - 9x^3y^3$

$= 6 \times (-1)^4(2)^2 - 9 \times (-1)^3 \times (2)^3$

$= 6 \times 1 \times 4 + 9 \times 1 \times 8$

$= 24 + 72 = 96$

$\therefore$ L.H.S. = R.H.S.

4. $\frac{2}{3}x^3y^3(3x - 15y) = \left(\frac{2}{3} \times 3\right)x^4y^3 - \left(\frac{2}{3} \times 15\right)x^3y^4$

$= 2x^4y^3 - 10x^3y^4$

To verify: if $x = 2$, $y = -1$

L.H.S. $= \frac{2}{3}x^3y^3(3x - 15y)$

$= \frac{2}{3} \times (2)^3(-1)^3\ [3 \times 2 - 15 \times (-1)]$

$= -\frac{2}{3} \times 8 \times 1 \times (6 + 15)$

$= -\frac{2}{3} \times 8 \times 1 \times 21 = -112$

R.H.S. $= 2x^4y^3 - 10x^3y^4$

$= 2 \times (2)^4 \times (-1)^3 - 10(2)^3 \times (-1)^4$

$= -2 \times 16 \times 1 - 10 \times 8 \times 1$

$= -32 - 80 = -112$

$\therefore$ L.H.S. = R.H.S.

5. (*i*) $5a(2a - 3c + b) - 2a(a - 3b + 5c)$

$= 10a^2 - 15ac + 5ab - 2a^2 + 6ab - 10ac$

$= (10a^2 - 2a^2) + (5ab + 6ab) + (-15ac - 10ac)$

$= 8a^2 + 11ab - 25ac$

(*ii*) $3(x + y)(x - y) + 3z(2y + z) - [3x(x - y - z) + 3y(x - y + z) + 3z(x + y - z)]$

$= 3x^2 - 3y^2 + 6yz + 3z^2 - 3x^2 + 3xy + 3xz$

$- 3xy + 3y^2 - 3yz - 3xz - 3yz + 3z^2$

$= (3x^2 - 3x^2) + (-3y^2 + 3y^2) + (3z^2 + 3z^2)$

$+ (6yz - 3y^2 - 3yz) + (3xy - 3xy) + (3xz - 3xz)$

$= 0 + 0 + 6z^2 + 0 + 0 + 0 = 6z^2$

6. (*i*) $a(a - b) + b(c - a) + c(b - a)$

$= a^2 - ab + bc - ab + bc - ac$

$= a^2 - 2ab + 2bc - ac$

(*ii*) $2x(y - 2x + z) + 3y(2z + x + x^2)$

$= 2xy - 4x^2 + 2xz + 6yz + 3yx + 3x^2y$

$= -4x^2 + 5xy + 3x^2y + 6yz + 2xz$

EXERCISE 6.4

1. $(5a + 3b)(3a - 5b) = 5a(3a - 5b) + 3b(3a - 5b)$
$= 15a^2 - 25ab + 9ab - 15b^2$
$= 15a^2 - 15b^2 - 16ab$
$= 15a^2 - 16ab - 15b^2$

2. $(2a - b)(4a - b) = 2a(4a - b) - b(4a - b)$
$= 8a^2 - 2ab - 4ab + b^2$
$= 8a^2 - 6ab + b^2$

3. $(3a^2 + 2b^2)(2a^2 - 5b^2) = 3a^2(2a^2 - 5b^2) + 2b^2(2a^2 - 5b^2)$
$= 6a^4 - 15a^2b^2 + 4a^2b^2 - 10b^4$
$= 6a^4 - 11a^2b^2 - 10b^4$

4. $2(x^2 - 2)(x^2 + 7) = 2x^2(x^2 + 7) - 4(x^2 + 7)$
$= 2x^4 + 14x^2 - 4x^2 - 28$
$= 2x^4 - 10x^2 - 28$

5. $(2 - 5x)(7x^2 + 3) = 2(7x^2 + 3) - 5x(7x^2 + 3)$
$= 14x^2 + 6 - 35x^3 - 15x$
$= - 35x^3 + 14x^2 - 15x + 6$

6. $(x^2 - a^2)(x - a) = x^2(x - a) - a^2(x - a)$
$= x^3 - ax^2 - a^2x + a^3$
$= x^3 - ax^2 - a^2x + a^3$

7. $(2x^2 + 7x)(4x - 3) = 2x^2(4x - 3) + 7x(4x - 3)$
$= 8x^3 - 6x^2 + 28x^2 - 21x$
$= 8x^3 + 22x^2 - 21x$

8. $(3p^2 + 2q^2)(2p^2 - 3q^2) = 3p^2(2p^2 - 3q^2) + 2q^2(2p^2 - 3q^2)$
$= 6p^4 - 9p^2q^2 + 4p^2q^2 - 6q^4$
$= 6p^4 - 5p^2q^2 - 6q^4$

9. $\left(x^2 + \frac{1}{x^2}\right)\left(x^2 + \frac{1}{x^2}\right) = x^2\left(x^2 + \frac{1}{x^2}\right) + \frac{1}{x^2}\left(x^2 + \frac{1}{x^2}\right)$
$= x^4 + 1 + 1 + \frac{1}{x^4}$
$= x^4 + \frac{1}{x^4} + 2$

10. $(x^2 - y^2)(x + 2y) = x^2(x + 2y) - y^2(x + 2y)$
$= x^3 + 2x^2y - xy^2 - 2y^3$

11. (*i*) $(2x + 7y)(x - 3y) - (9x + 4)(3x - 2)$
$= 2x(x - 3y) + 7y(x - 3y) - 9x(3x - 2) - 4(3x - 2)$
$= 2x^2 - 6xy + 7xy - 21y^2 - 27x^2 + 18x - 12x + 8$
$= (2x^2 - 27x^2) + (- 6xy + 7xy) + (- 21y^2) + 18x - 12x + 8$
$= - 25x^2 + xy - 21y^2 + 6x + 8$

(*ii*) $(5x^2 - y^2)(x^2 + 3y^2) + (2x^2 + y^2)(x^2 - 3y^2)$
$= 5x^2(x^2 + 3y^2) - y^2(x^2 + 3y^2) + 2x^2(x^2 - 3y^2) + y^2(x^2 - 3y^2)$
$= 5x^4 + 15x^2y^2 - x^2y^2 - 3y^4 + 2x^4 - 6x^2y^2 + x^2y^2 - 3y^4$
$= 7x^4 + 9x^2y^2 - 6y^4$
$= 7x^4 - 6y^4 + 9x^2y^2$

12. $(x^2 - 3)(x + 3) + 9$
$= x^2(x + 3) - 3(x + 3) + 9$
$= x^3 + 3x^2 - 3x - 9 + 9$
$= x^3 + 3x^2 - 3x$

13. $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$
$= a(c - d) + b(c - d) + a(c + d) - b(c + d) + 2ac + 2bd$
$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$
$= (ac + ac + 2ac) + (- ad + ad) + (bc - bc) + (- bd - bd + 2bd)$
$= 4ac + 0 + 0 + 0 = 4ac$

14. $(a + b - c)(a - b + c)$
$= a(a - b + c) + b(a - b + c) - c(a - b + c)$
$= a^2 - ab + ac + ab - b^2 + bc - ac + bc - c^2$
$= a^2 - b^2 - c^2 + 2bc$

15. $(a - b)(a^2 + ab + b^2)$
$= a(a^2 + ab + b^2) - b(a^2 + ab + b^2)$
$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$
$= a^3 - b^3$

16. $(3x^2 - x + 5)(x^2 - x - 1)$

$$
\begin{array}{rrrrrl}
 & (3x^2 & - x & + 5) & & \\
\times & (x^2 & - x & - 1) & & \\
\hline
-3x^4 & - x^3 & + 5x^2 & & & \text{(Multiplying by } x^2\text{)} \\
 & - 3x^3 & + x^2 & - 5x & & \text{(Multiplying by } - x\text{)} \\
 & & - 3x^2 & - x & - 5 & \text{(Multiplying by } - 1\text{)} \\
\hline
3x^4 & - 4x^3 & + 3x^2 & - 4x & - 5 & \\
\hline
\end{array}
$$

17.
$$
\begin{array}{rrrrr}
 & x^2 & - 8x & + 5 & \\
\times & (x^2 & - 3x & + 2) & \\
\hline
x^4 & - 8x^3 & + 5x^2 & & \\
 & - 3x^3 & + 24x^2 & - 15x & \\
 & & 2x^2 & - 16x & + 10 \\
\hline
x^4 & - 11x^3 & + 31x^2 & - 31x & + 10 \\
\hline
\end{array}
$$

18.
$$
\begin{array}{rrrr}
(x^2 & - 5x & + 7) & \\
 & \times (2x & - 5) & \\
\hline
2x^3 & - 10x^2 & + 14x & \\
 & - 5x^2 & + 25x & - 35 \\
\hline
2x^3 & - 15x^2 & + 39x & - 35 \\
\hline
\end{array}
$$

19.

$$\begin{array}{rrrr} 2x^2 & -\,9x & +\,6 & \\ & \times\,(3x & +\,4) & \\ \hline 6x^3 & -\,27x^2 & +\,18x & \\ & +\,8x^2 & -\,36x & +\,24 \\ \hline 6x^3 & -\,19x^2 & -\,18x & +\,24 \\ \hline \end{array}$$

20. (*i*) $(3x - 2)(2x + 3) - (3x + 5)(x - 1)$

$= 3x(2x + 3) - 2(2x + 3) - 3x(x - 1) - 5(x - 1)$

$= 6x^2 + 9x - 4x - 6 - 3x^2 + 3x - 5x + 5$

$= 3x^2 + 3x - 1$

(*ii*) $(x^2 - 2x + 3)(x - 5) - (x - 2)(2x^2 - 5x + 4)$

$= x^2(x - 5) - 2x(x - 5) + 3(x - 5) - 2x^2(x - 2) + 5x(x - 2) - 4(x - 2)$

$= x^3 - 5x^2 - 2x^2 + 10x + 3x - 15 - 2x^3 + 4x^2 + 5x^2 - 10x - 4x + 8$

$= - x^3 + 2x^2 - x - 7$

21. (*i*) $(x + 1)(2x - 1)(x + 3)$

$[x(2x - 1) + 1(2x - 1)](x + 3)$

$= (2x^2 - x + 2x - 1)(x + 3)$

$= (2x^2 + x - 1)(x + 3)$

$= 2x^2(x + 3) + x(x + 3) - 1(x + 3)$

$= 2x^3 + 6x^2 + x^2 + 3x - x - 3$

$= 2x^3 + 7x^2 + 2x - 3$

(*ii*) $a^2(a + 2b)b^2(a - b)$

$= a^2b^2(a + 2b)(a - b)$

$= a^2b^2[a(a - b) + 2b(a - b)]$

$= a^2b^2[a^2 - ab + 2ab - 2b^2]$

$= a^2b^2[a^2 + ab - 2b^2]$

$= a^4b^2 + a^3b^3 - 2a^2b^4$

22.

$$\begin{array}{rrr} (2x & -\,5y & \\ \times\;(x & -\,3y) & \\ \hline 2x^2 & -\,5xy & \\ & -\,6xy & +\,15y^2 \\ \hline 2x^2 & -\,11xy & +\,15y^2 \\ \hline \end{array}$$

$\therefore\ (2x - 5y)(x - 3y) = 2x^2 - 11xy + 15y^2$

To verify: $x = - 1$, $y = 2$

L.H.S. $= (2x - 5y)(x - 3y)$

$= [2 \times (- 1) - 5 \times 2] \times [- 1 - 3 \times 2]$

$= (- 2 - 10)(- 1 - 6)$

$= - 12 \times (- 7) = 84$

R.H.S. $= 2x^2 - 11xy + 15y^2$

$= 2 \times (- 1)^2 - 11 \times (- 1) \times 2 + 15(2)^2$

$= 2 + 22 + 60 = 84$

$\therefore$ L.H.S. = R.H.S.

23.

$$\begin{array}{rrr} (x^2y^2 & -\,2) & \\ \times\;(1 & -\,2x^2y^2) & \\ \hline x^2y^2 & -\,2 & \\ -\,2x^4y^4 & +\,4x^2y^2 & \\ \hline -\,2x^4y^4 & +\,5x^2y^2 & -\,2 \\ \hline \end{array}$$

$\therefore\ (x^2y^2 - 2)(1 - 2x^2y^2) = 5x^2y^2 - 2x^4y^4 - 2$

To verify: $x = - 1$, $y = 2$.

L.H.S. $= (x^2y^2 - 2)(1 - 2x^2y^2)$

$= [(- 1)^2 \times (2)^2 - 2][1 - 2 \times (- 1)^2 \times 2^2]$

$= (1 \times 4 - 2)(1 - 2 \times 4)$

$= 2 \times (- 7) = - 14$

R.H.S. $= 5x^2y^2 - 2x^4y^4 - 2$

$= 5 \times (- 1)^2 \times 2^2 - 2 \times (- 1)^4 \times 2^4 - 2$

$= 5 \times 1 \times 4 - 2 \times 1 \times 16 - 2$

$= 20 - 32 - 2 = - 14$

$\therefore$ L.H.S. = R.H.S.

24. $\left(x^2 - \dfrac{1}{2x^2}\right)\left(\dfrac{1}{3}x^2 + \dfrac{1}{x^2}\right)$

$= x^2\left(\dfrac{1}{3}x^2 + \dfrac{1}{x^2}\right) - \dfrac{1}{2x^2}\left(\dfrac{1}{3}x^2 + \dfrac{1}{x^2}\right)$

$= \dfrac{1}{3}x^4 + 1 - \dfrac{1}{6} - \dfrac{1}{2x^4}$

$= \dfrac{1}{3}x^4 - \dfrac{1}{2x^4} + \dfrac{5}{6}$

To verify: $x = - 1$

L.H.S. $= \left(x^2 - \dfrac{1}{2x^2}\right)\left(\dfrac{1}{3}x^2 + \dfrac{1}{x^2}\right)$

$= \left[(-1)^2 - \dfrac{1}{2(-1)^2}\right]\left[\dfrac{1}{3}(-1)^2 + \dfrac{1}{(-1)^2}\right]$

$= \left(1 - \dfrac{1}{2}\right)\left(\dfrac{1}{3} + 1\right)$

$= \dfrac{1}{2} \times \dfrac{4}{3} = \dfrac{2}{3}$

R.H.S. $= \dfrac{1}{3}x^4 - \dfrac{1}{2x^4} + \dfrac{5}{6}$

$= \dfrac{1}{3}(-1)^4 - \dfrac{1}{2 \times (-1)^4} + \dfrac{5}{6}$

$= \dfrac{1}{3} - \dfrac{1}{2} + \dfrac{5}{6} = \dfrac{2 - 3 + 5}{6}$

$= \dfrac{4}{6} = \dfrac{2}{3}$

$\therefore$ L.H.S. = R.H.S.

25. $\left(\frac{3}{5}x - \frac{y}{2}\right)\left(\frac{5}{3}x + 6y\right) = \frac{3}{5}x\left(\frac{5}{3}x + 6y\right) - \frac{y}{2}\left(\frac{5}{3}x + 6y\right)$

$$= x^2 + \frac{18}{5}xy - \frac{5}{6}xy - 3y^2$$

$$= x^2 - 3y^2 + \left(\frac{18}{5} - \frac{5}{6}\right)xy$$

$$= x^2 + \frac{83}{30}xy - 3y^2$$

To verify: $x = -1$, $y = 2$

$$\text{L.H.S.} = \left(\frac{3}{5}x - \frac{y}{2}\right)\left(\frac{5}{3}x + 6y\right)$$

$$= \left[\frac{3}{5}(-1) - \frac{2}{2}\right]\left[\frac{5}{3}\times(-1) + 6\times 2\right]$$

$$= \left(-\frac{3}{5} - 1\right)\left(-\frac{5}{3} + 12\right)$$

$$= \frac{-8}{5}\times\frac{-31}{3} = \frac{-248}{15}$$

$$\text{R.H.S.} = x^2 - 3y^2 + \frac{83}{30}\times xy$$

$$= (-1)^2 - 3\times(2)^2 + \frac{83}{30}\times(-1)\times 2$$

$$= 1 - 12 - \frac{83}{15} = -11 - \frac{83}{15}$$

$$= \frac{-165-83}{15} = \frac{-248}{15}$$

L.H.S. = R.H.S.

EXERCISE 6.5

1. (*i*) $(a^2 + b^2)^2 = (a^2)^2 + 2(a^2)(b^2) + (b^2)^2$

$[\because\ (a + b)^2 = a^2 + 2ab + b^2]$

$= a^4 + b^4 + 2a^2b^2$

(*ii*) $(xy + 2z)^2 = (xy)^2 + 2(xy)(2z) + (2z)^2$

$[\because\ (a + b)^2 = a^2 + 2ab + b^2]$

$= x^2y^2 + 4xyz + 4z^2$

(*iii*) $(a^2 - b^2)^2 = (a^2)^2 - 2(a^2)(b^2) + (b^2)^2$

$[\because\ (a - b)^2 = a^2 - 2ab + b^2]$

$= a^4 - 2a^2b^2 + b^4$

(*iv*) $(4x - 5)^2 = (4x)^2 - 2(4x)(5) + (5)^2$

$[\because\ (a - b)^2 = a^2 - 2ab + b^2]$

$= 16x^2 - 40x + 25$

(*v*) $(7x - 4)^2 = (7x)^2 - 2(7x)(4) + (4)^2$

$[\because\ (a - b)^2 = a^2 - 2ab + b^2]$

$= 49x^2 - 56x + 16$

(*vi*) $(4l + 5m)^2 = (4l)^2 + 2(4l)(5m) + (5m)^2$

$[\because\ (a + b)^2 = a^2 + 2ab + b^2]$

$= 16l^2 + 40lm + 25m^2$

2. (*i*) $(x + 5)(x + 5) = (x + 5)^2$

$= x^2 + 2\cdot x\cdot 5 + 5^2$

$[\because\ (a + b)^2 = a^2 + 2ab + b^2]$

$= x^2 + 10x + 25$

(*ii*) $(3x - 7)(3x - 7) = (3x - 7)^2$

$= (3x)^2 - 2(3x)7 + (7)^2$

$[\because\ (a - b)^2 = a^2 - 2ab + b^2]$

$= 9x^2 - 42x + 49$

(*iii*) $(a - c)(-a + c) = -(a - c)(a - c)$

$= -(a - c)^2$

$= -[a^2 - 2ac + c^2]$

$= -a^2 + 2ac - c^2$

(*iv*) $\left(2a - \frac{1}{2}\right)\left(2a + \frac{1}{2}\right) = (2a)^2 - \left(\frac{1}{2}\right)^2$

$[\because\ (a - b)(a + b) = a^2 - b^2]$

$= 4a^2 - \frac{1}{4}$

(*v*) $\left(\frac{2}{3}m + \frac{3}{2}n\right)\left(\frac{2}{3}m - \frac{3}{2}n\right) = \left(\frac{2}{3}m\right)^2 - \left(\frac{3}{2}n\right)^2$

$[\because\ (a - b)(a + b) = a^2 - b^2]$

$= \frac{4}{9}m^2 - \frac{9}{4}n^2$

(*vi*) $(2x + 5y)(2x - 5y) = (2x)^2 - (5y)^2$

$[\because\ (a - b)(a + b) = a^2 - b^2]$

$= 4x^2 - 25y^2$

3. (*i*) $(x + 4)(x + 3) = x^2 + (4 + 3)x + 12$

$[\because\ (x + a)(x + b) = x^2 + (a + b)x + ab]$

$= x^2 + 7x + 12$

(*ii*) $(3x + 5y)(3x + 2y) = (3x)^2 + (5y + 2y)\cdot 3x + 10y^2$

$= 9x^2 + 21xy + 10y^2$

(*iii*) $(2x + 3)(2x + 5) = (2x)^2 + (3 + 5)\ 2x + 3\times 5$

$= 4x^2 + 16x + 15$

(*iv*) $(4p + 3q)(4p + 7q) = (4p)^2 + (3q + 7q)\cdot 4p + 21q^2$

$[\because\ (x + a)(x + b) = x^2 + (a + b)x + ab]$

$= 16p^2 + 40pq + 21q^2$

(*v*) $(a + 3b)(a + 5b) = a^2 + (3b + 5b)a + 15b^2$

$[\because\ (x + a)(x + b) = x^2 + (a + b)x + ab]$

$= a^2 + 8ab + 15b^2$

(*vi*) $(7x + 9)(7x + 8) = (7x)^2 + (9 + 8)\cdot 7x + 72$

$[\because\ (x + a)(x + b) = x^2 + (a + b)x + ab]$

$= 49x^2 + 119x + 72$

4. (*i*) $(2x - 1)^2 - (x - 1)^2$

$= [(2x)^2 - 2(2x)1 + (1)^2] - [x^2 - 2x + 1]$

$[\because\ (a - b)^2 = a^2 - 2ab + b^2]$

$= 4x^2 - 4x + 1 - x^2 + 2x - 1$

$= 3x^2 - 2x + 0 = 3x^2 - 2x$

(*ii*) $(7 + 5x)(7 + 5x) - (2x - 3)(2x - 3)$

$= (7 + 5x)^2 - (2x - 3)^2$

$= [(7 + 5x) - (2x - 3)]\,[(7 + 5x) + (2x - 3)]$

$- [\because\ (a - b)(a + b) = a^2 - b^2]$

$= (10 + 3x)\,(4 + 7x)$

$= 21x^2 + 82x + 40$

(*iii*) $(6x + 3)(6x - 3) = (6x)^2 - (3)^2$

$[\because\ (a + b)(a - b) = a^2 - b^2]$

$= 36x^2 - 9$

(*iv*) $(2m - 3n)(2m + 3n) + (2m + 3n)^2$

$= [(2m)^2 - (3n)^2] + (2m + 3n)^2$

$[\because\ (a + b)(a - b) = a^2 - b^2]$

$= (4m^2 - 9n^2) + 4m^2 + 12mn + 9n^2$

$[\because\ (a + b)^2 = a^2 + 2ab + b^2]$

$= (4m^2 + 4m^2) + (-9n^2 + 9n^2) + 12\ mn$

$= 8m^2 + 12mn$

(*v*) $(4a + 5b)(4a + 7b) - (3a + 4b)(3a + 6b)$

$= (4a)^2 + (5b + 7b)\cdot 4a + (5b^2)$

$- [(3a)^2 + (4b + 6b)\cdot 3a + 24b^2]$

$[\because\ (x + a)(x + b) = x^2 + (a + b)x + ab^2]$

$= 16a^2 + 48ab + 35b^2 - 9a^2 - 30ab - 24b^2$

$= 7a^2 - 18ab + 11b^2$

$= 7a^2 + 18ab + 11b^2$

5. We have,

$(a + b)^2 = a^2 + 2ab + b^2$

$\Rightarrow\quad (a + b)^2 = (a^2 + b^2) + 2ab$

$\Rightarrow\quad (a + b)^2 = 100 + 2 \times 48$

$(\because\ a^2 + b^2 = 100 \text{ and } ab = 48)$

$= 100 + 96 = 196$

$(a + b) = \sqrt{196} = 14$

$a + b = 14$

6. We have,

$(a + b)^2 = a^2 + 2ab + b^2$

$\Rightarrow\quad (a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow\quad (9)^2 = (a^2 + b^2) + 2 \times 4$

$\Rightarrow\quad 81 = (a^2 + b^2) + 8$

$\Rightarrow\quad a^2 + b^2 = 81 - 8 = 73$

Hence, $a^2 + b^2 = 73$

7. We have,

$(a - b)^2 = a^2 - 2ab + b^2$

$\Rightarrow\quad (a - b)^2 = (a^2 + b^2) - 2ab$

$\Rightarrow\quad (5)^2 = 49 - 2ab$

$\Rightarrow\quad 25 - 49 = -2ab$

$\Rightarrow\quad -24 = -2ab$

$ab = \frac{24}{2} = 12$

Hence, $ab = 12$

8. Given that

$$x + \frac{1}{x} = 5$$

Squaring on both sides, we get

$$\left(x + \frac{1}{x}\right)^2 = (5)^2$$

$\Rightarrow\quad x^2 + \frac{1}{x^2} + 2 = 25 \quad [\because\ (a + b)^2 = a^2 + 2ab + b^2]$

$\Rightarrow\quad x^2 + \frac{1}{x^2} = 25 - 2$

$\Rightarrow\quad x^2 + \frac{1}{x^2} = 23$

Again, squaring on both sides, we get,

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (23)^2$$

$$x^4 + \frac{1}{x^4} + 2 = 529$$

$$x^4 + \frac{1}{x^4} = 529 - 2$$

$\Rightarrow\quad x^4 + \frac{1}{x^4} = 527$

9. Given that $x - \frac{1}{x} = 3$,

Squaring on both sides, we get

$$\left(x - \frac{1}{x}\right)^2 = 3^2$$

$\Rightarrow\quad x^2 + \frac{1}{x^2} - 2 = 9$

$$x^2 + \frac{1}{x^2} = 9 + 2$$

$$x^2 + \frac{1}{x^2} = 11$$

Again, squaring on both sides, we get,

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (11)^2$$

$$x^4 + \frac{1}{x^4} + 2 = 121$$

$$x^4 + \frac{1}{x^4} = 121 - 2 = 119$$

Hence, $x^4 + \frac{1}{x^4} = 119$

10. (*i*) $(x + 1)(x - 1)(x^2 + 1)$

$= (x^2 - 1)(x^2 + 1)$

$[\because\ (a + b)(a - b) = a^2 - b^2]$

$= (x^2)^2 - (1)^2$

$[\because\ (a + b)(a - b) = a^2 - b^2]$

$= x^4 - 1$

(*ii*) $(x + 2)(x - 2)(x^2 + 4)$

$= [x^2 - (2)^2](x^2 + 4)$

$[\because \ (a + b)(a - b) = a^2 - b^2]$

$= (x^2 - 4)(x^2 + 4)$

$= (x^2)^2 - (4)^2$

$[\because \ (a - b)(a + b) = a^2 - b^2]$

$= x^4 - 16$

(*iii*) $(2m - 3)(2m + 3)(4m^2 + 9)$

$= [(2m)^2 - (3)^2](4m^2 + 9)$

$[\because \ (a - b)(a + b) = a^2 - b^2]$

$= (4m^2 - 9)(4m^2 + 9)$

$= (4m^2)^2 - (9)^2$

$[\because \ (a - b)(a + b) = a^2 - b^2]$

$= 16m^4 - 81$

(*iv*) $(3x + 2y)(3x - 2y)(9x^2 + 4y^2)$

$= [(3x)^2 - (2y)^2](9x^2 + 4y^2)$

$[\because \ (a - b)(a + b) = a^2 - b^2]$

$= (9x^2 - 4y^2)(9x^2 + 4y^2)$

$= (9x^2)^2 - (4y^2)^2$

$[\because \ (a - b)(a + b) = a^2 - b^2]$

$= 81x^4 - 16y^4$

11. We have

$(x - y)^2 = x^2 + y^2 - 2xy$

$\Rightarrow \quad (8)^2 = (x^2 + y^2) - 2 \times 5$

$\Rightarrow \quad 64 = (x^2 + y^2) - 10$

$\Rightarrow \quad x^2 + y^2 = 64 + 10 = 74$

$\Rightarrow \quad x^2 + y^2 = 74$

12. (*i*) $(81)^2 = (80 + 1)^2$

$= (80)^2 + 2 \times 80 \times 1 + (1)^2$

$[\because \ (a + b)^2 = a^2 + b^2 + 2ab]$

$= 6400 + 160 + 1 = 6561$

(*ii*) $(198)^2 = (200 - 2)^2$

$= (200)^2 - 2 \times 200 \times 2 + (2)^2$

$[\because \ (a + b^2) = a^2 - 2ab + b^2]$

$= 40000 - 800 + 4 = 39204$

(*iii*) $165 \times 155 = (160 + 5)(160 - 5)$

$= (160)^2 - (5)^2$

$[\because \ (a + b)(a - b) = a^2 - b^2]$

$= 25600 - 25 = 25575$

(*iv*) $(7.25)^2 - (2.75)^2 = (7.25 - 2.75) \times (7.25 + 2.75)$

$= 4.50 \times 10.00 = 45$

(*v*) $(5.6)^2 - (4.4)^2 = (5.6 - 4.4)(5.6 + 4.4)$

$[\because \ (a - b)(a + b) = a^2 - b^2]$

$= 1.2 \times 10.0 = 12$

13. (*i*) $\dfrac{176 \times 176 - 124 \times 124}{52} = \dfrac{(176)^2 - (124)^2}{52}$

$= \dfrac{(176 - 124)(176 + 124)}{52}$

$[\because \ a^2 - b^2 = (a - b)(a + b)]$

$= \dfrac{52 \times 300}{52} = 300$

(*ii*) $\dfrac{324 \times 324 - 276 \times 276}{48} = \dfrac{(324)^2 - (276)^2}{48}$

$= \dfrac{(324 - 276)(324 + 276)}{48}$

$= \dfrac{48 \times 600}{48}$

$= 600$

14. $\left(x - \dfrac{1}{x}\right)^2 = x^2 + \dfrac{1}{x^2} - 2 \times x \times \dfrac{1}{x}$

$\left(x - \dfrac{1}{x}\right)^2 = \left(x^2 + \dfrac{1}{x^2}\right) - 2$

$\left(x - \dfrac{1}{x}\right)^2 = 27 - 2$

$\left(x - \dfrac{1}{x}\right)^2 = 25$

$x - \dfrac{1}{x} = \sqrt{25}$

$x - \dfrac{1}{x} = 5$

15. $\dfrac{36 \times 36 - 24 \times 24}{60} = \dfrac{(36)^2 - (24)^2}{60}$

$= \dfrac{(36 - 24) - (36 + 24)}{60}$

$= \dfrac{12 \times 60}{60} = 12$

16. (*i*) $3p = (28)^2 - (23)^2$

$\Rightarrow \quad 3p = (28 - 23) \times (28 + 23)$

$[\because \ a^2 - b^2 = (a - b)(a + b)]$

$\Rightarrow \quad 3p = 5 \times 51$

$\Rightarrow \quad p = \dfrac{5 \times 51}{3} = 5 \times 17$

$p = 85$

(*ii*) $7p = (25 \times 25) - (24 \times 24)$

$= (25)^2 - (24)^2$

$= (25 - 24)(25 + 24)$

$7p = 1 \times 49$

$[\because \ a^2 - b^2 = (a - b)(a + b)]$

$p = \dfrac{49}{7} = 7$

$p = 7$

17. $\left(x^2+\frac{1}{x^2}\right)^2 = \left(x^4+\frac{1}{x^4}\right)+2\times x^2\times\frac{1}{x^2}$

$$\left(x^2+\frac{1}{x^2}\right)^2 = 119 + 2$$

$$\left(x^2+\frac{1}{x^2}\right)^2 = 121$$

$$x^2+\frac{1}{x^2} = \sqrt{121} = 11$$

Now,

$$\left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2\times x\times\frac{1}{x}$$

$$= x^2+\frac{1}{x^2}-2$$

$$\left(x-\frac{1}{x}\right)^2 = 11 - 2 = 9$$

$$x-\frac{1}{x} = \sqrt{9} = 3$$

Hence, $x-\frac{1}{x} = 3$

18. (*i*) $36x^2 + 49y^2 + 84xy = (6x)^2 + (7y)^2 + 2(6x)(7y)$

$= (6x)^2 + 2(6x)(7y) + (7y)^2 = (6x + 7y)^2$

$[\because \ (a+b)^2 = a^2 + 2ab + b^2]$

Substituting the value of $x = \frac{1}{4}$ and $y = \frac{2}{7}$

$$= \left(6\times\frac{1}{4}+7\times\frac{2}{7}\right)^2 = \left(\frac{3}{2}+2\right)^2$$

$$= \left(\frac{7}{2}\right)^2 = \frac{49}{4}$$

(*ii*) $16x^2 + 25y^2 + 40xy$

$= (4x)^2 + (5y)^2 + 2 \times (4x)(5y)$

Substituting the value $x = \frac{1}{4}$ and $y = \frac{1}{5}$, we get

$$= \left(4\times\frac{1}{4}\right)^2+\left(5\times\frac{1}{5}\right)^2+2\times\left(4\times\frac{1}{4}\right)\times\left(5\times\frac{1}{5}\right)$$

$= 1 + 1 + 2 \times 1 \times 1 = 4$

(*iii*) $16m^2 + 36n^2 + 48mn = (4m)^2 + (6n)^2 + 2(4m)(6n)$

$= (4m + 6n)^2$

$[\because \ (a+b)^2 = a^2 + 2ab + b^2]$

Substituting the value $m = \frac{1}{2}$ and $n = \frac{1}{3}$, we get

$$= \left[\left(4\times\frac{1}{2}\right)+\left(6\times\frac{1}{3}\right)\right]^2 = (2+2)^2 = (4)^2 = 16$$

19. (*i*) $\frac{(89)^2-(11)^2}{156} = \frac{(89-11)(89+11)}{156}$

$$= \frac{\cancel{78}^{1}\times\cancel{100}^{50}}{\cancel{156}_{2}} = 50$$

(*ii*) $(12.9)^2 - (7.1)^2 = (12.9 - 7.1)(12.9 + 7.1)$

$[\because \ a^2 - b^2 = (a-b)(a+b)]$

$= 5.8 \times 20 = 116$

20. (*i*) $\frac{(6a-5b)^2-(6a+5b)^2}{ab} = -120$

$$\text{L.H.S.} = \frac{(6a-5b)^2-(6a+5b)^2}{ab}$$

$$= \frac{[(6a-5b)-(6a+5b)][(6a-5b)+(6a+5b)]}{ab}$$

$[\because \ a^2 - b^2 = (a-b)(a+b)]$

$$= \frac{(6a-5b-6a-5b)(6a-5b+6a+5b)}{ab}$$

$$= \frac{(-10b)\times(12a)}{ab} = \frac{-120ab}{ab}$$

$= -120 =$ R.H.S.

$\therefore$ L.H.S. = R.H.S.

Hence proved

(*ii*) $\frac{(4ab+3cd)^2-(4ab-3cd)^2}{48} = abcd$

$$\text{L.H.S.} = \frac{(4ab+3cd)^2-(4ab-3cd)^2}{48}$$

$$= \frac{[(4ab+3cd)-(4ab-3cd)][(4ab+3cd)+(4ab-3cd)]}{48}$$

$$= \frac{[(4ab+3cd-4ab+3cd)(4ab+3cd+4ab-3cd)]}{48}$$

$$= \frac{(6cd)\times(8ab)}{48}$$

$$= \frac{48abcd}{48} = abcd = \text{R.H.S.}$$

L.H.S. = R.H.S.

Hence, Proved

(*iii*) $\frac{(2a+b)^2-(2a-b)^2}{ab} = 8$

$$\text{L.H.S.} = \frac{(2a+b)^2-(2a-b)^2}{ab}$$

$$= \frac{[(2a+b)-(2a-b)][(2a+b)+(2a-b)]}{ab}$$

$$= \frac{(2a+b-2a+b)(2a+b+2a-b)}{ab}$$

$$= \frac{(2b)\times(4a)}{ab} = \frac{8(ab)}{ab} = 8 = \text{R.H.S.}$$

Hence, L.H.S. = R.H.S.

(iv) $\dfrac{(7pqr-5)^2-(7pqr+5)^2}{(pqr+1)^2-(pqr-1)^2} = -35$

L.H.S. $= \dfrac{(7pqr-5)^2-(7pqr+5)^2}{(pqr+1)^2-(pqr-1)^2}$

$= \dfrac{[(7pqr-5)-(7pqr+5)][(7pqr-5)+(7pqr+5)]}{[(pqr+1)-(pqr-1)][(pqr+1)+(pqr-1)]}$

$[\because\ a^2 - b^2 = (a-b)(a+b)]$

$= \dfrac{(7pqr-5-7pqr-5)(7pqr-5+7pqr+5)}{(pqr+1-pqr+1)(pqr+1+pqr-1)}$

$= \dfrac{(-10)(14\,pqr)}{(2)(2pqr)} = \dfrac{-140pqr}{4pqr}$

$= -35 =$ R.H.S.

Hence, L.H.S. = R.H.S.

MULTIPLE CHOICE QUESTIONS

1. $6a^2b^2 - 4ab$

when $a = 2$, $b = -1$

$= 6 \times (2)^2 \times (-1)^2 - 4 \times 2 \times (-1)$

$= 6 \times 4 \times (1) - 4 \times 2 \times (-1)$

$= 24 + 8 = 32$

Hence, option (c) is correct.

2. $145 \times 145 - 144 \times 144 = (145)^2 - (144)^2$

$= (145 - 144)(145 + 144)$

$= 1 \times 289 = 289$

Hence, option (a) is correct.

3. $x - \dfrac{1}{x} = 4$

Squaring on both sides, we get

$$\left(x - \frac{1}{x}\right)^2 = (4)^2$$

$$x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = 16$$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} - 2 = 16$$

$$x^2 + \frac{1}{x^2} = 16 + 2 = 18$$

Hence, option (b) is correct.

4. $\dfrac{(7.95)^2 - (2.05)^2}{(7.95 - 2.05)} = \dfrac{(7.95-2.05)(7.95+2.05)}{(7.95-2.05)}$

$= 7.95 + 2.05 = 10.00$

Hence, option (c) is correct.

5. $a + b = 7$

Squaring on both sides, we get

$(a + b)^2 = (7)^2$

$a^2 + b^2 + 2ab = 49$

$a^2 + b^2 + 2 \times 6 = 49$

$a^2 + b^2 = 49 - 12$

$a^2 + b^2 = 37$

Hence, option (c) is correct.

6. $(-a^2b)(abc) = -a^{2+1} \times b^{1+1} - c$

$= -a^3b^2c$

Hence, option (c) is correct.

7. $8x = (45)^2 - (43)^2$

$= (45 - 43)(45 + 43)$

$\Rightarrow \quad 8x = 2 \times 88$

$\Rightarrow \quad x = \dfrac{2 \times 88}{8} = 22$

Hence, option (d) is correct.

8. $1.71 \times 1.71 - 0.29 \times 0.29$

$= (1.71)^2 - (0.29)^2$

$= (1.71 - 0.29)(1.71 + 0.29)$

$= 1.42 \times 2$

$= 2.84$

Hence, option (b) is correct.

9. If $x - y = 7$

Squaring on both sides, we get

$(x - y)^2 = (7)^2$

$(x - y)^2 = 49$

$x^2 + y^2 - 2xy = 49$

$x^2 + y^2 - (2 \times 9) = 49$

$x^2 + y^2 = 49 + 18 = 67$

Hence, option (a) is correct.

10. $16x^2 - 24xy + 9y^2$

$= (4x)^2 - 2(4x)(3y) + (3y)^2$

$= (4x - 3y)^2$

$[\because\ (a - b)^2 = a^2 - 2ab + b^2]$

Substituting the values of x and y, we get

$= \left(4 \times \dfrac{1}{4} - 3 \times \dfrac{1}{3}\right)^2$

$= (1 - 1)^2 = (0)^2 = 0$

Hence, option (a) is correct.

11. $\left(x + \dfrac{1}{x}\right)\left(x - \dfrac{1}{x}\right) = x^2 - \left(\dfrac{1}{x}\right)^2$

$= x^2 - \dfrac{1}{x^2}$

$= x^2 - x^{-2}$

Hence, option (b) is correct.

12. $\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$

$\left(x+\frac{1}{x}\right)^2 = 23 + 2 \qquad \left(\because\ x^2+\frac{1}{x^2}=23\right)$

$\left(x+\frac{1}{x}\right)^2 = 25$

$\left(x+\frac{1}{x}\right) = \sqrt{25} = 5$

Hence, option (d) is correct.

MENTAL MATHS CORNER

1. x^2y^2 is a monomial. **(True)**
2. $\frac{2x^2-x}{x}$ is a polynomial. **(True)**
3. $(2m^2 - 3n^2)(3m^2 + 2n^2) = 6m^4 - 6n^4$. **(False)**

   $\because\ (2m^2 - 3n^2)(3m^2 + 2n^2)$
   $= 6m^4 + 4m^2n^2 - 9m^2n^2 - 6n^4$
   $= 6m^4 - 5m^2n^2 - 6n^4$
4. The term of expression having no literal factor is called a constant term. **(True)**
5. The symbol which takes various values is called a variable. **(True)**
6. $ab(a - b) = a^2b - ab^2$ **(True)**
7. The degree of a polynomial $6x^3 + 7x^2y^2 - 6y^2 + 9$ is 3. **(False)**
8. $\frac{a^4-b^4}{a-b} = (a + b)(a^2 + b^2)$ **(True)**

   $= \frac{(a^2-b^2)(a^2+b^2)}{(a-b)} = \frac{(a-b)(a+b)(a^2+b^2)}{(a-b)}$

   $= (a + b)(a^2 + b^2)$

REVIEW EXERCISE

1. We have

$\left(x^2+\frac{1}{x^2}\right)^2 = x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2}$

$[\because\ (a + b)^2 = (a^2 + 2ab + b^2)]$

$\left(x^2+\frac{1}{x^2}\right)^2 = x^4+\frac{1}{x^4}+2$

$\left(x^2+\frac{1}{x^2}\right)^2 = 6239 + 2 \qquad \left(\because\ x^4+\frac{1}{x^4}=6239\right)$

$\left(x^2+\frac{1}{x^2}\right)^2 = 6241$

$\left(x^2+\frac{1}{x^2}\right) = \sqrt{6241}$

$\left(x^2+\frac{1}{x^2}\right) = 79$

Now,

$\left(x+\frac{1}{x}\right)^2 = \left(x^2+\frac{1}{x^2}\right)+2\times x\times\frac{1}{x}$

$= 79 + 2 = 81$

$x+\frac{1}{x} = \sqrt{81} = 9$

2. We have

$(x - y)^2 = x^2 - 2xy + y^2$

$= (x^2 + y^2) - 2xy$

$(x - y)^2 = 29 - 2 \times 2$

$(\because\ x^2 + y^2 = 29$ and $xy = 2)$

$(x - y)^2 = 29 - 4$

$(x - y) = \sqrt{25} = 5$

3. Sum of $3x - x^2 + 7$ and $- 2x - 3 + 5x^2$

$$\begin{array}{rrr} -x^2 & +3x & +7 \\ 5x^2 & -2x & -3 \\ \hline +4x^2 & +x & +4 \\ \hline \end{array}$$

(On adding)

Now,

$7 - (4x^2 + x + 4) = 7 - 4x^2 - x - 4$

$= 3 - x - 4x^2$

4. $\frac{196\times196-104\times104}{92} = \frac{(196)^2-(104)^2}{92}$

$= \frac{(196-104)(196+104)}{92}$

$= \frac{92\times300}{92} = 300$

5. $(x - y)(x + y) + (y - z)(y + z) + (z - x)(z + x) = 0$

L.H.S. $= (x - y)(x + y) + (y - z)(y + z) + (z - x)(z + x)$

$= (x^2 - y^2) + (y^2 - z^2) + (z^2 - x^2)$

$[\because\ (a - b)(a + b) = a^2 - b^2]$

$= x^2 - y^2 + y^2 - z^2 + z^2 - x^2$

$= 0$

= R.H.S.

$\therefore$ L.H.S. = R.H.S.

Hence proved.

6. (i) $(103)^2 - (101)^2 = (103 - 101)(103 + 101)$

$\because\ a^2 - b^2 = (a - b)(a + b)$

$= 2 \times 204 = 408$

(ii) $(998)^2 - (2)^2 = (998 - 2)(998 + 2)$

$\because\ a^2 - b^2 = (a - b)(a + b)$

$= 996 \times 1000 = 996000$

7. (*i*) $(-3x^2) \times (4a^2x^5) \times (3a^3x^4) = [(-3) \times 4 \times 3]a^5x^{11}$

$= -36a^5x^{11}$

(*ii*) $\frac{3}{5}xy \times \frac{5}{6}x^2y \times \frac{2}{5}xy^3 = \left(\frac{3}{5} \times \frac{5}{6} \times \frac{2}{5}\right)(xy \times x^2y \times xy^3)$

$= \frac{1}{5}x^4y^5$

8. We have,

$3x + 2y = 12$

Squaring on both sides, we have

$(3x + 2y)^2 = (12)^2$

$9x^2 + 4y^2 + 12xy = 144$

$9x^2 + 4y^2 + 12 \times 6 = 144 \qquad (\because \quad xy = 6)$

$9x^2 + 4y^2 = 144 - 72 = 72$

Hence, $9x^2 + 4y^2 = 72$

9. $15x = (50)^2 - (40)^2$

$15x = (50 - 40)(50 + 40)$

$[\because \quad a^2 - b^2 = (a - b)\ (a + b)]$

$= 10 \times 90$

$15x = 900$

$x = \frac{900}{15} = 60$

$x = 60$

10. $(5x + 3)(x - 1)(2x - 3)$

$= [5x(x - 1) + 3(x - 1)](2x - 3)$

$= (5x^2 - 5x + 3x - 3)(2x - 3)$

$= (5x^2 - 2x - 3)(2x - 3)$

$= 2x\ (5x^2 - 2x - 3) - 3(5x^2 - 2x - 3)$

$= 10x^3 - 4x^2 - 6x - 15x^2 + 6x + 9$

$= 10x^3 - 19x^2 + 9$

HOTS QUESTIONS

1. Double of $\left(x - \frac{2}{x}\right) = 2\left(x - \frac{2}{x}\right)$

Triple of $\left(x + \frac{2}{x}\right) = 3\left(x + \frac{2}{x}\right)$

Product of $2\left(x - \frac{2}{x}\right)$ and $3\left(x + \frac{2}{x}\right)$

$= 2\left(x - \frac{2}{x}\right) \times 3\left(x + \frac{2}{x}\right)$

$= 6\left(x - \frac{2}{x}\right)\left(x + \frac{2}{x}\right)$

$= 6\left[x^2 - \left(\frac{2}{x}\right)^2\right]$

$[\because \quad a^2 - b^2 = (a - b)(a + b)]$

$= 6\left(x^2 - \frac{4}{x^2}\right)$

2. Sum of $x^2 - 4x + 7$ and $2x^2 + 5x - 9$

$$\begin{array}{r} x^2 - 4x + 7 \\ 2x^2 + 5x - 9 \\ \hline 3x^2 + x \quad - 2 \\ \hline \end{array}$$

Now, the required expression

$= 0 - (3x^2 + x - 2)$

$= -3x^2 - x + 2$

Hence, the required expression is $-3x^2 - x + 2$.

3. We have,

$\left(x + \frac{1}{x}\right) = \sqrt{5}$

Squaring on both sides, we get

$\left(x + \frac{1}{x}\right)^2 = (\sqrt{5})^2$

$x^2 + \frac{1}{x^2} + 2 = 5$

$x^2 + \frac{1}{x^2} = 5 - 2$

$x^2 + \frac{1}{x^2} = 3$

Again, squaring on both sides, we get

$\left(x^2 + \frac{1}{x^2}\right)^2 = (3)^2$

$x^4 + \frac{1}{x^4} + 2 = 9$

$\Rightarrow \qquad x^4 + \frac{1}{x^4} = 9 - 2 = 7$

Hence $\qquad x^4 + \frac{1}{x^4} = 7$

Puzzle

Let x and y be the two unequal number of coins.

Difference between two numbers $= (x - y)$

Difference between the squares of the two numbers $= (x^2 - y^2)$

As per condition,

$(x^2 - y^2) = 47(x - y)$

$\cancel{(x - y)}(x + y) = 47\cancel{(x - y)}$

$(x + y) = 47$

Hence, total number of coins = 47.